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A NEW METHOD TO EVALUATE THE MINIMUM VOLUME INCLUDING RADIATING/SCATTERING SYSTEMS BY MEANS OF SUPPORTING CONES

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Abstract - A new approach to estimate the support of a source and/or scattering system from radiated and/or scattered (single frequency and single view) field data on a given observation plane is presented for the 3D geometry. The approach is based on the recently introduced local field properties PSSC and LSC and represents a significant and nontrivial extension of the method to detect the convex hull of a radiating/scattering system recently introduced by some of the authors in the 2D geometry. The volume estimated by the proposed technique contains the support of the object, it is *not larger* than the convex hull of the object and can be also a not-convex set. The numerical analysis confirms that the approach can also describe the non convexity of the support of the radiating system.

1. INTRODUCTION

An effective technique to estimate the (minimum) region containing a radiating/scattering system from field data represents a crucial tool in many practical instances involving inverse electromagnetic problems.

First of all, in some applications, the attention is devoted only to the detection of the presence and to the localization of radiating/scattering inhomogeneities (e.g., demining, security, archaeological survey, buildings diagnostics, etc.), without paying attention to the inversion of their electromagnetic properties. In such cases, effective methods demand for approaches providing only the information of interest, i.e. the volume enclosing the radiators. Accordingly, those techniques which afford the complete Inverse Source (IS) or Inverse Scattering (ISC) [5, 8] problem by finding also the electromagnetic properties of the object does not represent the best answer to the problem at hand. In fact, apart from the theoretical aspects concerning the non uniqueness and ill-posedness of such problems, the use of the full inversion algorithms is often unfeasible in such a practical application, due to the serious difficulties related to the ill-conditioning, the occurrence of false solutions and the high computational burden.

On the other side, in those cases which require the successful and reliable solution of the complete IS or ISC problem, the estimation of the support of the radiating system represents a first crucial step. In fact, the support estimate provides to the complete inversion algorithms an a priori information which can significantly contribute to the well posedness of the problem as well as to the reduction of the occurrence of false solutions and of the computational effort.

As a matter of fact, when information about the support of the scattering system is unavailable, the solution of the inverse problem requires the retrieval of a large number of unknowns. To face such a difficulty, some approaches attempt to get additional information about the object by considering multi-view and/or multi-frequency scattering illuminations (often without a robust criterion on how to quantify and select such additional information), with a (serious) increase of computer time and storage requirements. Moreover, some approaches can be successfully applied only by considering small searching regions which, in many instances, are assumed to be a priori known.

Thus, a technique able to estimate the support of a radiating system is very useful in practice either as a stand alone technique or as a pre-processing step in the solution of complete IS or ISC problems.

It must be stressed that the retrieval of the geometrical information of interest should be based on field properties related only to the geometry of the radiating system.

Recently, referring to the 2D geometry, a new approach to estimate the region containing the source/scattering systems from field data on a given observation line has been introduced and experimentally tested in [2, 4]. The technique is based on the PSSC and LSC concepts [3] and is able to estimate a set close to (but *not smaller* than) the convex hull containing the radiating system.

The aim of this paper is to present the nontrivial extension of the method to the 3D scalar case. The approach is essentially new, if compared to the 2D one, and determine a volume *not larger* than the convex hull and containing the support of the object. To this end, the new concept of *supporting cones* is introduced and the

Minimum Volume enclosing the radiating system as given by the *Supporting Cones* (SCMV) is defined and estimated.

The technique has been extensively tested on numerically synthesized field data and some of the main results are discussed in Section 4. They prove the effectiveness of the approach and show that the SCMV is qualified to describe, in some instances, the non convexity of the radiating system.

2. STATEMENT OF THE PROBLEM AND SOLUTION STRATEGY

Let us consider, in a reference system Oxyz, an observation plane Π, with equation z=d. Let us denote with E(P) the electromagnetic field disturbance (3D-scalar case) radiated by the object and collected at the observation point $P \in \Pi$. We aussume E(P) known and the radiating system (source and/or scattering) ***S*** contained in the half-space z<d.

The problem to be considered here amounts to estimate the support of ***S*** from the knowledge of the field E on Π. The exp(jωt) time dependence will be assumed and dropped later on. The three dimensional homogeneous space will be denoted with $\mathbb{R}^3$.

Given a point P on Π, we define as a supporting cone Σ(P) (associated to ***S***) any cone with vertex at P whose boundary contains points of ***S*** and whose complement, say $\Sigma^C(P) = \mathbb{R}^3 \setminus \Sigma(P)$, includes ***S*** completely. A coordinate supporting cone is any supporting cone with axis parallel to one of the three coordinate axes. We define the SCMV as the intersection of all the sets $\Sigma^C(P)$ obtained by considering all the coordinate supporting cones with vertex at P, as P moves in Π.

It can be shown that the SCMV is a set including ***S*** and enclosed in the convex hull of ***S*** [1].

As such, the SCMV is a set smaller than the convex hull estimated for the 2D geometry in [2], wherein the estimation of the minimum cones including ***S*,** with vertex at the observation points, is exploited.

As a consequence the SCMV can be also a not-convex set and thus, it could describe also the non convexity of ***S*** (as, for instance, holes).

Obviously, for a fixed plane Π, the SCMV depends on the choice of the x and y axis, although the above properties just outlined are independent on such a choice. In principle, this dependence could be exploited to improve quality of the estimation of the support of ***S***.

According to these considerations, the approach presented here is based on the two following steps:

a) to estimate the coordinate supporting cones corresponding to a discrete (convenient) set of points P_i, i = 1,…,N, on Π, from the knowledge of the radiated field E;
b) to find the intersection set between all the complement sets $\Sigma^C(P_i)$, i =1,…, N, of the supporting cones evaluated at point a).

Obviously, the choice of the points P_i is a matter of convenience and results from the trade off between the computational cost of the algorithm and the accuracy of the estimation. It can be also upgraded during the estimation process to enlighten some details of the SCMV.

The estimation of the supporting cones involved at point a) is performed by exploiting the Point Source Spectral Content (PSSC) and the Local Spectral Content (LSC) concepts recently introduced in [2, 3] and extended to the 3D case. They describe local properties of the radiated field, strictly related to the geometry of the radiating system and easily valuable from the field on Π by means of a *direct* and *stable* approach.

3. THE DETERMINATION OF THE SUPPORTING CONES BY USING THE PSSC AND THE LSC CONCEPTS

Given the observation point P($\underline{r}$), $\underline{r} = x\hat{i}_x + y\hat{i}_y + d\hat{i}_z = \underline{r}_t + d\hat{i}_z$, we denote with Q($\underline{r}_t$) its orthogonal projection on the z = 0 plane.

According to [2, 3], we introduce the *reduced* (with respect to the point Q) as:

$$F(\underline{r}) = E(\underline{r}) \exp\left(j\beta \left| \underline{r} - \underline{r}_t \right| \right) \tag{1}$$

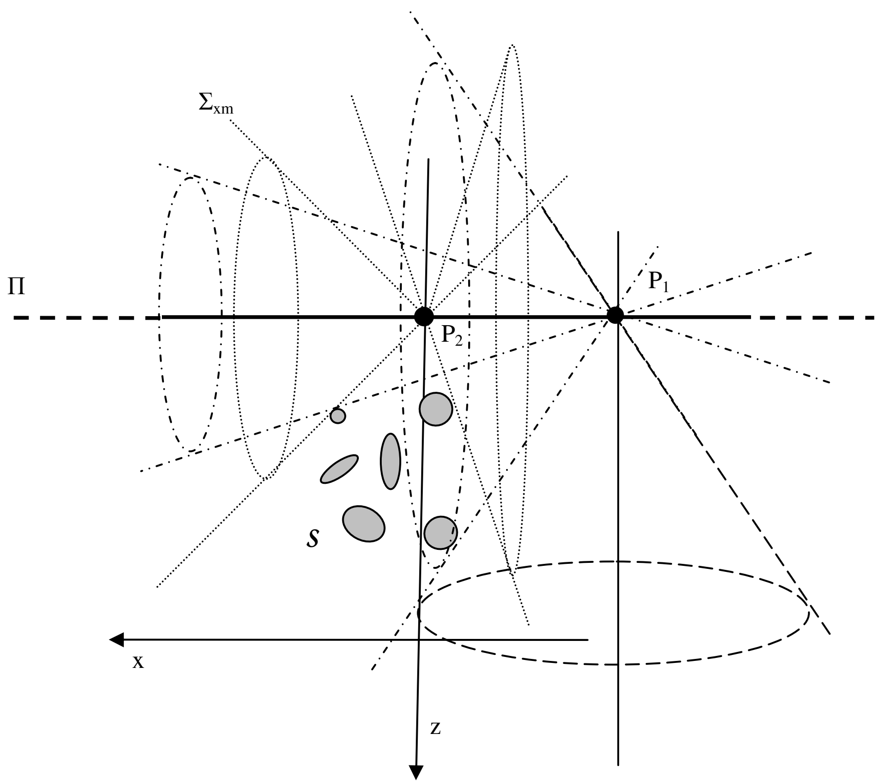

Figure 1. The supporting cones found by the algorithm. Cones with axis along x with vertex at P_1 in case A (-.-) and with vertex at P_2 in case B (…). Cones with axis along z with vertex at P_1 (--).

By denoting with $\tilde{F}(\underline{k}, \underline{r})$ the local Fourier transform of F [6], i.e. the *sliding-window spectrum* of F, we have:

$$\tilde{F}(\underline{k}, \underline{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(\underline{r}'_t - \underline{r}_t) F(\underline{r}') \exp(-j\,\underline{k} \bullet \underline{r}'_t)\, d\underline{r}'_t = \int_{S}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(\underline{r}'_t - \underline{r}_t) \frac{N(\underline{r}', \underline{\bar{r}})}{|\underline{r}' - \underline{\bar{r}}|} \exp\left(j\left(\psi(\underline{r}', \underline{\bar{r}}) - \underline{k} \bullet \underline{r}'_t\right)\right) d\underline{r}'_t\, d\underline{\bar{r}} \qquad (2)$$

where $\underline{r}' = \underline{r}'_t + d\hat{i}_z = x'\hat{i}_x + y'\hat{i}_y + d\hat{i}_z$, $\beta = 2\pi/\lambda$, λ is the wavelength, $\underline{k} = k_x\hat{i}_x + k_y\hat{i}_y$ is conjugate to $\underline{r}_t$, $N(\underline{r}', \underline{\bar{r}})$ is a slowly varying factor related to the Green function [7], $g(\underline{r}_t)$ is the windowing function, and $\psi(\underline{r}, \underline{\bar{r}}) = \beta\left(|\underline{r} - \underline{r}_t| - |\underline{r}, \underline{\bar{r}}|\right)$.

For each $\underline{\bar{r}}$ and for $\beta \rightarrow \infty$, the double integral in equation (2) can be asymptotically evaluated by the stationary phase method [7]. As discussed also in [2, 3], by neglecting end points and/or slope diffraction effects [7] and by taking into account that the window function is significantly different from zero only in a neighbourhood of $\underline{r}$, we can say that only stationary points falling inside the small interval wherein the sliding window is different from zero are of interest. As a consequence, we can say that $\underline{\tilde{F}}(\underline{k}, \underline{r})$ is essentially different from zero only for those values of $\underline{k}$ that satisfy the relationship:

$$\underline{k} \approx \underline{h}(\underline{r}, \underline{\bar{r}}) = h_x(\underline{r}, \underline{\bar{r}}) i_x + h_y(\underline{r}, \underline{\bar{r}}) i_y \triangleq (\partial\psi(\underline{r}, \underline{\bar{r}})/\partial x) i_x + (\partial\psi(\underline{r}, \underline{\bar{r}})/\partial y) i_y \qquad (3)$$

In other words, for a given $\underline{\bar{r}}$, the functions h_x and h_y locate the position of a small neighborhood of the $\underline{k}$ plane wherein the local spectrum $\underline{\tilde{F}}(\underline{k}, \underline{r})$ due to a point-wise source at $\underline{\bar{r}}$ is significantly different from zero.

Such a property of the functions h_x and h_y suggest their name, Point Source Spectral Content (PSSC) of the field.

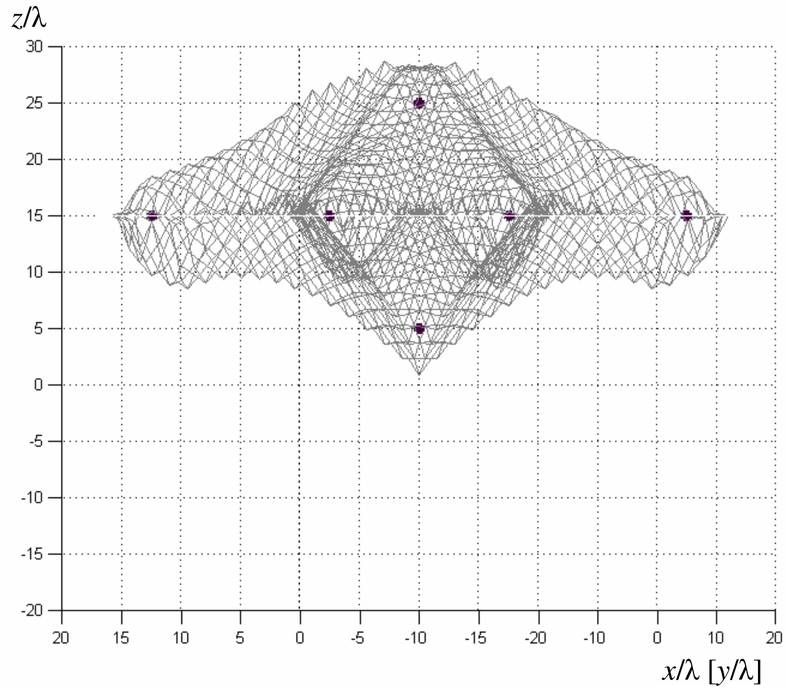

Figure 2. A view of the estimated SCMV (sources = black dots).

By varying $\underline{\bar{r}}$, the functions h_x and h_y define the region of the $\underline{k}$ plane wherein related to the spectral distribution of the reduced field, the LSC, is significantly different from zero [1-3].
In the case of our interest, it can be shown that:

$$h_x(\underline{r},\underline{\bar{r}}) = -\beta \hat{i}_x \bullet (\underline{r}-\underline{\bar{r}}) / |\underline{r}-\underline{\bar{r}}| \quad (4a)$$

$$h_y(\underline{r},\underline{\bar{r}}) = -\beta \hat{i}_y \bullet (\underline{r}-\underline{\bar{r}}) / |\underline{r}-\underline{\bar{r}}| \quad (4b)$$

i.e. $\underline{h}_x/\beta$ and $\underline{h}_y/\beta$ are given by the opposite of the x and y components of the unit ray from the source point $\underline{\bar{r}}$ to the observation point $\underline{r}$, respectively.

Given an observation point $P(\underline{r}_0)$, $\underline{r}_0 = x_0\hat{i}_x + y_0\hat{i}_y + d\hat{i}_z$ on Π, let us consider the determination of the supporting cones associated to the x axis and let us refer to Figure 1.

Two cases must be distinguished:

- case A: a single supporting cone departs from P since $\boldsymbol{S}$ fully belongs to only one of the half-spaces $x \le x_0$, $x \ge x_0$ (Figure 1);
- case B: a couple of supporting cones depart from P since $\boldsymbol{S}$ extends on both half-spaces (Figure 1).

Let us denote with $\boldsymbol{S}_{xm}$ and $\boldsymbol{S}_{xM}$ the set of points of $\boldsymbol{S}$ corresponding to the minimum and maximum value of $\underline{h}_x(\underline{r}_0,\underline{\bar{r}})$, $\underline{\bar{r}} \in \boldsymbol{S}$, say $\underline{h}_{xm}$ and $\underline{h}_{xM}$, respectively. $\boldsymbol{S}_{xm}$ and $\boldsymbol{S}_{xM}$ are made by the points of $\boldsymbol{S}$ corresponding to unit rays directed to $P(\underline{r}_0)$ making the minimum and the maximum angle with the x-axis, respectively.

When $\underline{h}_{xm} \le \underline{h}_{xM} \le 0$ or $0 \le \underline{h}_{xm} \le \underline{h}_{xM}$ the case A is of concern. The supporting cone passes through the points of $\boldsymbol{S}_{xm}$ or $\boldsymbol{S}_{xM}$ in the case $\underline{h}_{xM} \le 0$ or $0 \le \underline{h}_{xM}$, respectively.

When $\underline{h}_{xm} \le 0 \le \underline{h}_{xM}$, the case B is of concern and the two supporting cones are those passing through the points of $\boldsymbol{S}_{xm}$ and $\boldsymbol{S}_{xM}$, respectively (Figure 1).

Similar reasoning apply to the determination of the coordinate supporting cones associated to the y-axis.

To consider the supporting cones with axes parallel to the z-axis, we define $h_z(\underline{r},\overline{r}) = \sqrt{\underline{h}_x^2(\underline{r},\overline{r}) + \underline{h}_y^2(\underline{r},\overline{r})}$ and note that $h_z(\underline{r}_0,\ \overline{r})/\beta$ gives the z-component of the unit ray from the source point $\overline{r}$ to the observation point $\underline{r}_0$. Again, let us denote with S_{zm} the set of points of S corresponding to unit rays directed to P($\underline{r}_0$) making the minimum angle with the z-axis. Since the points of S_{zm} correspond to the minimum value of $h_z(\underline{r}_0, \overline{r}), \overline{r} \in S$, provided that this minimum is not equal to zero, it can be easily seen that the supporting cone associated to the z-axis is the one passing through the points of S_{zm} (Figure 1).

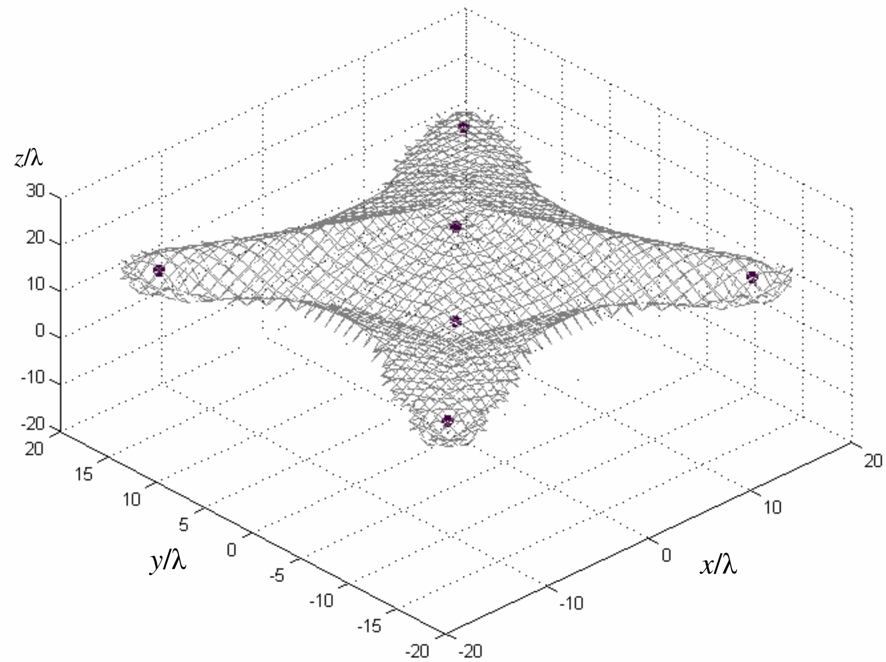

Figure 3. A view of the estimated SCMV (sources = black dots).

4. NUMERICAL ANALYSIS

The presented technique has been implemented in a numerical code and tested against simulated noiseless field data. In this communication we present the result of the procedure related to the case of a radiating object made by a set of six point-wise sources with coordinates, normalised to λ, given by (15 ,15 ,15) ,(-15, 15 ,15), (-15, -15 ,15), (15, -15, 15), (0, 0, 25), (0, 0, 5). The amplitudes and phases of the source excitation are randomly distributed between [0.5A, 1A] and [0, 2π], respectively. The observation plane Π with equation z = 30λ has been considered. The x-y coordinates of the points of Π exploited to estimate the support of the source are given by (10nλ, 10mλ) with n, m=-3,..,3. Three view of the obtained SCMV are shown under Figures 2-4. The cut of the SCMV with the plane z = 15λ is shown in Figure 5. The presented results clearly prove that the SCMV is included within the convex hull of the radiating system and confirm the ability of the approach to estimate a non convex volume enclosing the radiating system.

In authors experience, similar results can be obtained by processing noisy data.

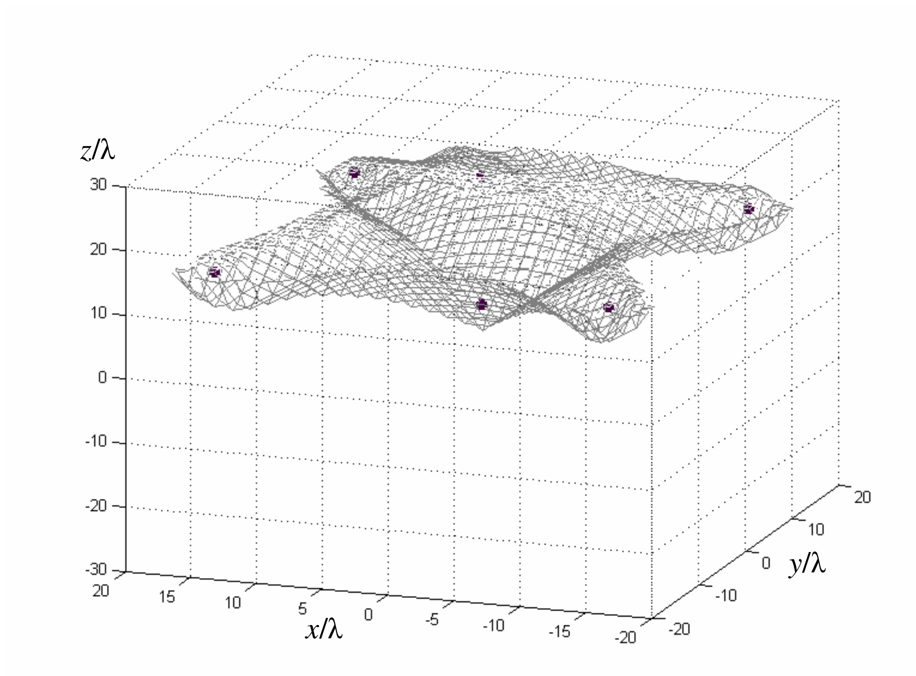

Figure 4. A view of the estimated SCMV (sources = black dots).

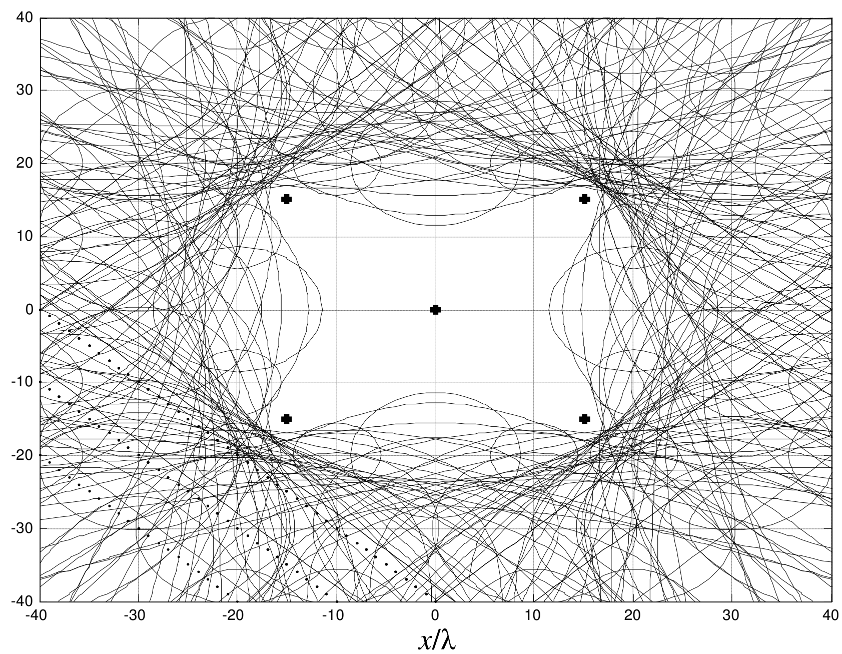

Figure 5. The z = 15λ cut of the estimated SCMV (sources = black dots).

5. CONCLUSIONS

A new method to evaluate the minimum volume including radiating/scattering systems by means of supporting cones working in the 3D scalar case for the planar scanning has been presented. The approach is able to provide an estimate the support of the object by determine a non convex (theoretically) volume containing the object and contained in the convex hull of the object: the *Minimum Volume* enclosing the radiating system as given by the *Supporting Cones* (SCMV). The method does not require the solution of the complete Inverse Problem. In fact, it is based on properties of the electromagnetic field, the PSSC and the LSC, which depend only on the geometry of the radiating system and can be determined from the observed field data by means of a simple, stable and direct approach exploiting the Local Fourier Transform. Numerical results validate the proposed approach and prove that the SCMV is qualified to describe, in some instances, the non convexity of the radiating system.

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